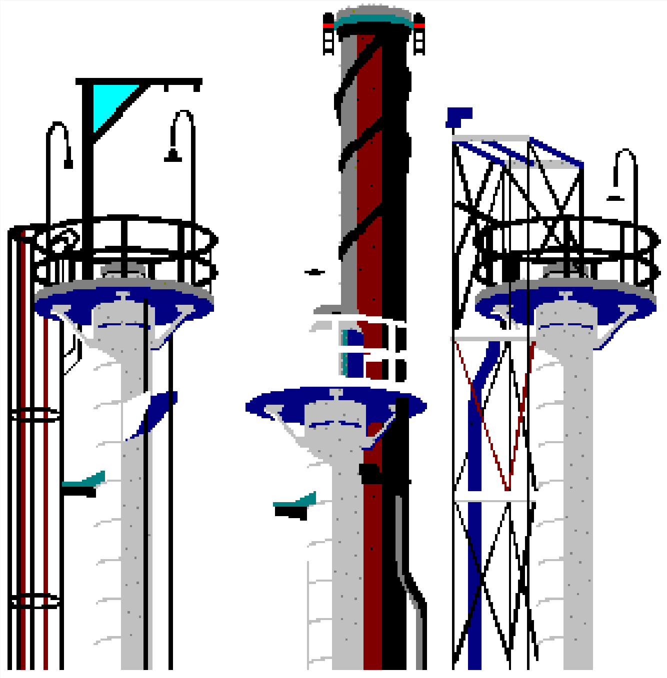
Chapter4 Principles of Steady-State Heat Transfer



이 광남

차원해석

- Buckingham π theorem
  ~ u개의 기본적 단위나 차원으로 주어진 q개의 양 또는 변수의 함수 관계는 (q-u)개의 독립적 군(π)으로 표현 할 수 있다.
- Buckingham method
  특별한 물리적 문제에 있어서 중요한 변수들을 나열한 다음에 Buckingham π theorem에 의해 변수를 결합하여 무차원 파라메터의 수를 결정

강제대류

- 상황~직경이 D인 파이프 속을 유체가 v로 흐르면서 (난류) 관 벽으로 열을 전달
- 변수~D, k ,μ, v, ρ, c_P, h ; (q=7)
- 기본단위 또는 차원~M, L, t, T ; (u=4)
- 무차원군 수(π수)=q-u=3

  $\pi_1=f(\pi_2, \pi_3)$

변수의 단위

$$h[W/m^2K = J/s\,m^2\,K = kg\,m^2/s^2sm^2\,K = kg/s^3\,K] = \frac{M}{t^3T}$$

$$D = L,\ \rho = \frac{M}{L^3},\ \mu = \frac{M}{Lt},\ v = \frac{L}{t}$$

$$k[W/m\,K = J/s\,m\,K = kg\,m^2/s^2sm\,K = kg\,m/s^3\,K] = \frac{ML}{t^3T}$$

$$c_p[J/kg\,K = kg\,m^2/s^2\,kg\,K = m^2/s^2\,K] = \frac{L^2}{t^2T}$$

$$\pi_1 = D^a k^b \mu^c v^d \rho$$

$$\pi_2 = D^a k^b \mu^c v^d c_p$$

$$\pi_3 = D^a k^b \mu^c v^d h$$

$$\pi_1 = D^a k^b \mu^c v^d \rho$$

$$[M^0 L^0 t^0 T^0] = L^a (\frac{ML}{t^3 T})^b (M / Lt)^c (L / t)^d (\frac{M}{L^3})$$

$$M: 0 = b + c + 1$$

$$L: 0 = a + b - c + d - 3$$

$$t: 0 = -3b - c - d$$

$$T: 0 = -b$$

$$b = 0, c = -1, d = 1, a = 1 \Rightarrow \pi_1 = D\mu^{-1}v\rho = \frac{\rho v D}{\mu}$$

같은 방법으로

$$\pi_2 = \frac{c_p \mu}{k},\ \pi_3 = \frac{hD}{k}$$

$$\frac{hD}{k} = f\left(\frac{\rho v D}{\mu}, \frac{c_p \mu}{k}\right) \Rightarrow Nu = f(\mathrm{Re}, \mathrm{Pr})$$

$$Nu = \frac{hD}{k} = \frac{(D/kA)}{(1/hA)} = \frac{\text{전도 열전달 저항}}{\text{대류 열전달 저항}} \ : Nusselt\ No$$

$$\mathrm{Re} = \frac{\rho v D}{\mu} = \frac{\rho v^2}{\mu (v/D)} = \frac{\text{관성력}}{\text{점성력}}$$

$$\mathrm{Pr} = \frac{c_p \mu}{k} = \frac{(\mu/\rho)}{(k/c_p \rho)} = \frac{momentum\ diffusivity}{thermal\ diffusivity} = \frac{\text{수력학적 경계층 두께}}{\text{열 경계층 두께}}$$

$: Prandtle\ No$

자연대류

- 상황~길이가 L인 수직 평면 벽에서 인접한 유체로의 자연 대류 열전달
- 변수~L, k, μ, c_P, h , β, ρ, g, ΔT ; (q=9)
- 기본단위 또는 차원~M, L, t, T ; (u=4)
- 무차원군 수(π수)=q-u=5

  $\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5)$

변수의 단위

$$L = L,\ h = \frac{M}{t^3T},\ \mu = \frac{M}{Lt},\ k = \frac{ML}{t^3T},\ c_p = \frac{L^2}{t^2T}$$

$$\rho = \frac{M}{L^3},\ \beta = \frac{1}{T},\ g = \frac{L}{t^2},\ \Delta T = T$$

$$\pi_1 = L^a \mu^b k^c g^d \rho \Rightarrow \pi_1 = \frac{L^3\rho^2 g}{\mu^2}$$

$$\pi_2 = L^a \mu^b k^c g^d c_p \Rightarrow \pi_2 = \frac{c_p\mu}{k} = \mathrm{Pr}$$

$$\pi_3 = L^a \mu^b k^c g^d \beta \Rightarrow \pi_3 = \frac{L\mu g\beta}{k}$$

$$\pi_4 = L^a \mu^b k^c g^d \Delta T \Rightarrow \pi_4 = \frac{k\Delta T}{L\mu g}$$

$$\pi_5 = L^a \mu^b k^c g^d h \Rightarrow \pi_5 = \frac{hL}{k} = Nu$$

$$\pi_1\pi_3\pi_4 = \frac{L^3\rho^2 g\beta\Delta T}{\mu^2} = Gr : Grashof\ No$$

$$Nu = f(Gr, \mathrm{Pr})$$

$$Gr = \frac{L^3\rho^2 g\beta\Delta T}{\mu^2} = \frac{\text{부력}}{\text{점성력}}$$

4.5 FORCED CONVECTION HEAT TRANSFER INSIDE PIPES

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4.5B Heat transfer coefficient for laminar flow inside a pipe

Sieder and Tate eqn (Re<2100)

$$Nu = \frac{h_a D}{k} = 1.86(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L})^{1/3}(\frac{\mu_b}{\mu_w})^{0.14}$$

D: pipe diameter $[m]$

L: pipe length $[m]$

μ_b: viscosity at bulk avg temp. $[Pa\,s]$

μ_w: viscosity at the wall temp.

c_p: heat capacity $[J/kg\,K]$

k: thermal conductivity $[W/m\,K]$

h_a: avg heat transfer coeff. $[W/m^2\,K]$

*모든 물성치는 유체의 평균온도에서 측정(μ_w제외)

$$q = h_a A \Delta T = h_a A \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2} = h_a A (T_w - \frac{T_{bi} + T_{bo}}{2})$$

4.5C Heat transfer coefficient for turbulent flow inside a pipe

- **Re>6000, 0.7<Pr<1600, L/D>60**

$$Nu = \frac{h_L D}{k} = 0.027 \mathrm{Re}^{0.8} \mathrm{Pr}^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

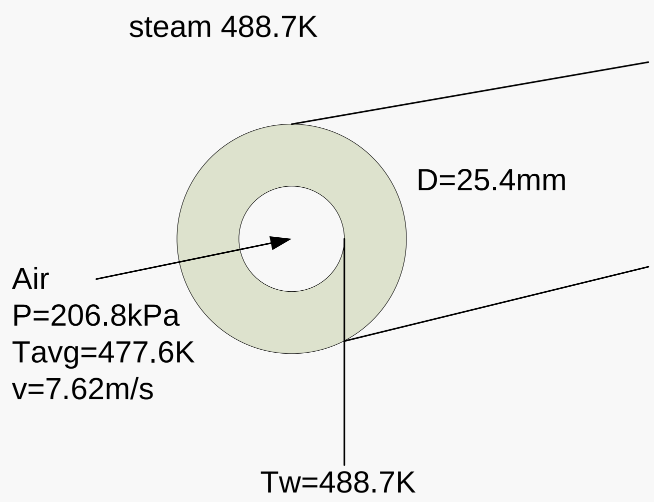
h_L : *heat transfer coeff. based on the* log *mean driving force*

$$\Delta T_{lm} \; (\Delta T_1 = T_w - T_{bi}, \; \Delta T_2 = T_w - T_{bo})[W / m^2 K]$$

*모든 물성치는 유체의 평균온도에서 측정(μ_w제외)

EXAMPLE 4.5-1. Heating of Air in Turbulent Flow

Air at 206.8 kPa and an average of 477.6 K is being heated as it flows through a tube of 25.4 mm inside diameter at a velocity of 7.62 m/s. The heating medium is 488.7 K steam condensing on the outside of the tube. Since the heat-transfer coefficient of condensing steam is several thousand $W/m^2 \cdot K$ and the resistance of the metal wall is very small, it will be assumed that the surface wall temperature of the metal in contact with the air is 488.7 K. Calculate the heat-transfer coefficient for an $L/D > 60$ and also the heat-transfer flux q/A.



$$T_{avg} = 477.6K$$

$$\mu_b = 2.6\times10^{-5}\,Pa\,s \quad from\,Table\,A.3-3$$

$$\Pr = 0.686,\ k = 0.03894[W/m\,K]$$

$\rho = \dfrac{PM}{RT}$ (공기가 $1atm$ 상태가 아니므로 $Table\,A.3-3$의 값을 바로 사용 못함)

$$= \frac{206.8kPa}{} \left| \frac{28.87g}{mol} \right| \frac{273.15K}{(101.325kPa)(22.4\ell/mol)} \left| \frac{}{477.6K} \right| \frac{kg}{1000g} \left| \frac{1000\ell}{m^3} \right.$$

$$= 1.509[kg/m^3]$$

$$\mathrm{Re} = \frac{\rho v D}{\mu} = \frac{1.509}{} \left| \frac{7.62}{} \right| \frac{0.0254}{} \left| \frac{}{2.6\times10^{-5}} \right. = 1.123\times10^4$$

$$Nu = \frac{h_L D}{k} = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}(\frac{\mu_b}{\mu_w})^{0.14}$$

μ_w ?

$477.6 \qquad 2.6\times10^{-5}\,Pa\,s$

$488.7 \quad \mu_w = 2.64\times10^{-5}$

$505.4 \qquad 2.71\times10^{-5}$

$$h_L = \frac{0.03894}{} \left| \frac{}{0.0254} \right| \frac{0.027}{} \left| \frac{(1.123\times10^4)^{0.8}}{} \right| \frac{0.686^{1/3}}{} \left| \frac{(2.6/2.64)^{0.14}}{} \right.$$

$$= \boxed{63.3[W/m^2\,K]}$$

$$q/A = h_L(T_w - T_{avg}) = 63.3\times(488.7-477.6) = \boxed{701.1[W/m^2]}$$

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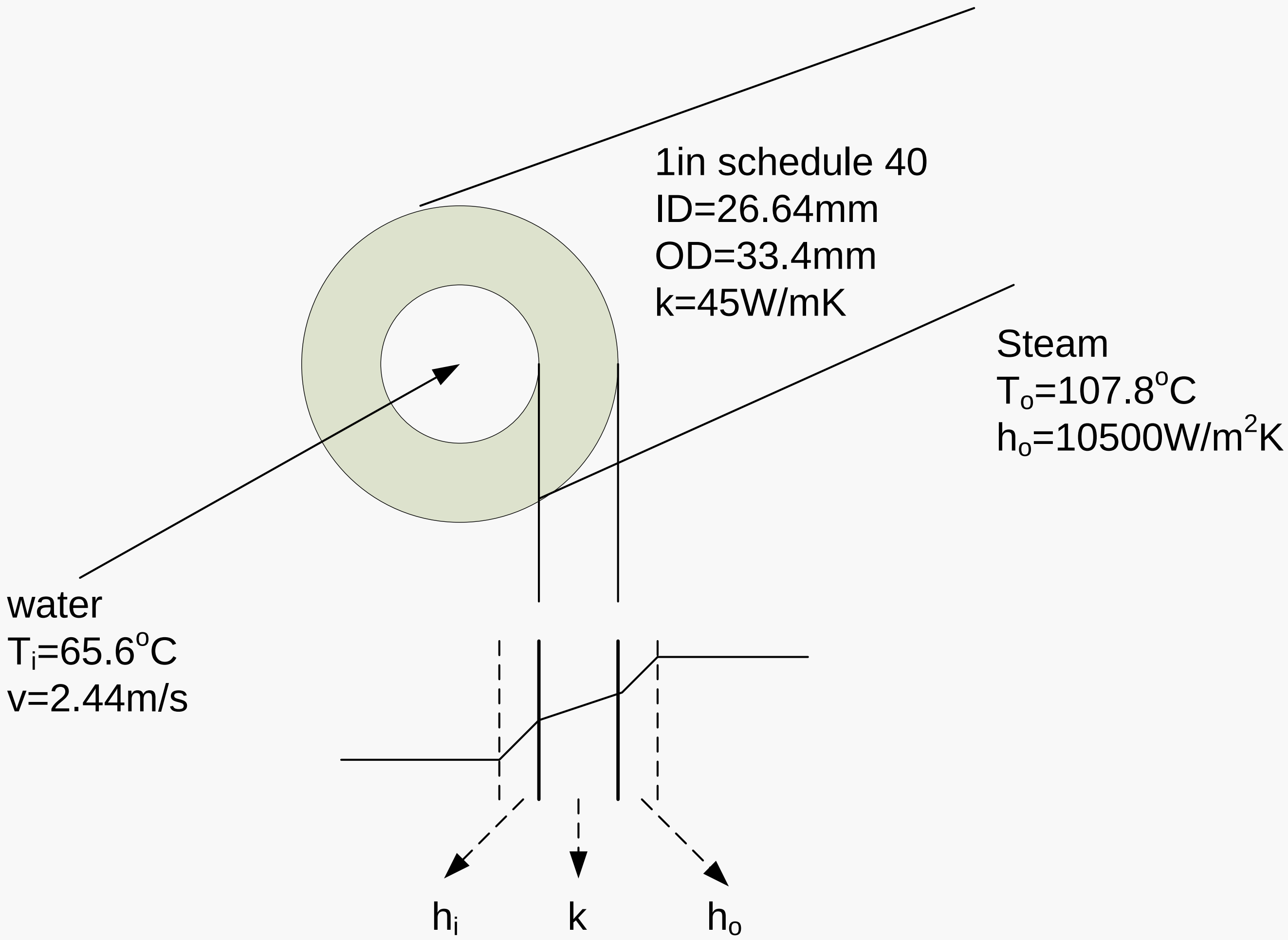
EXAMPLE 4.5-2. Water Heated by Steam and Trial-and-Error Solution

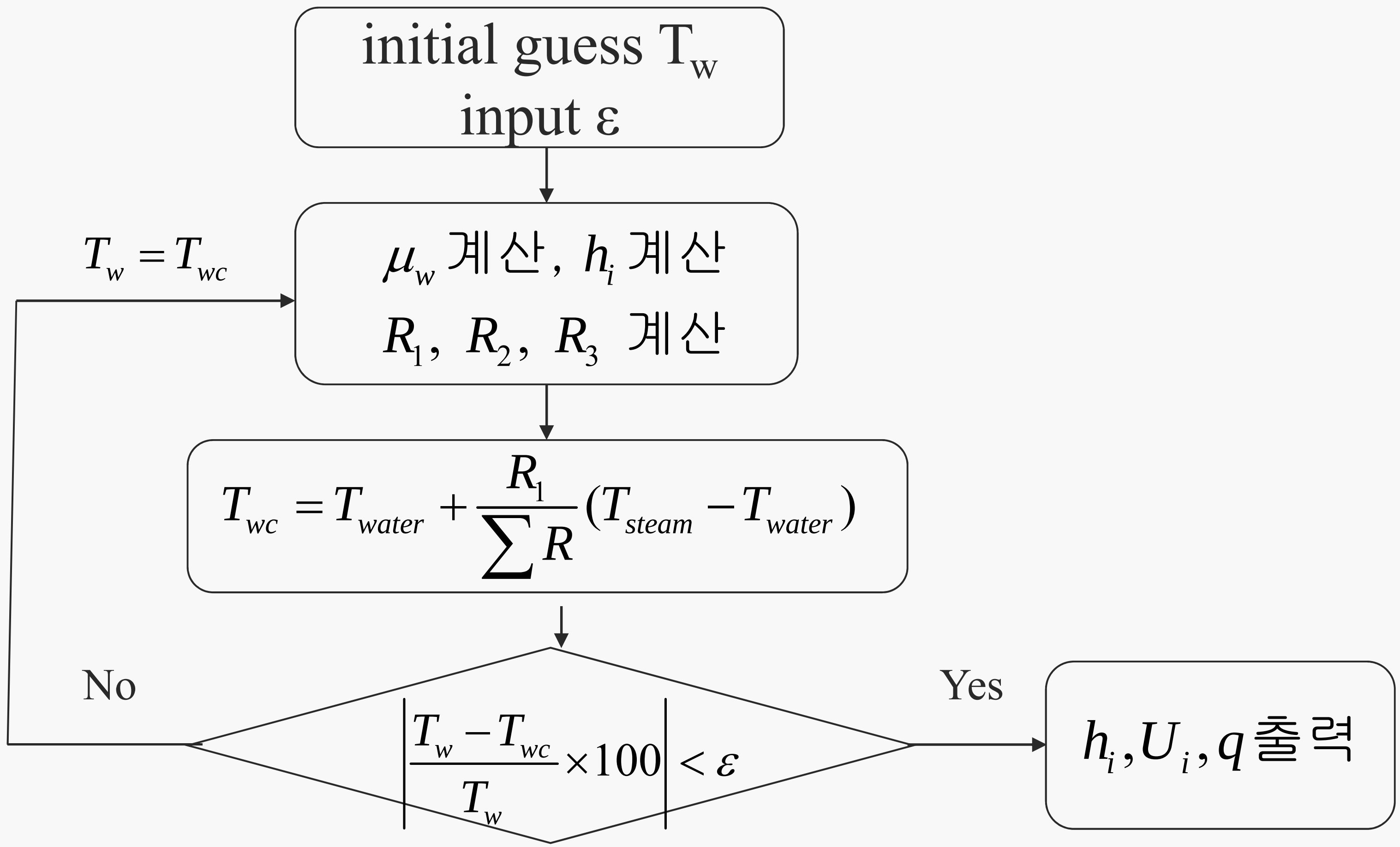
Water is flowing in a horizontal 1-in. schedule 40 steel pipe at an average temperature of 65.6°C and a velocity of 2.44 m/s. It is being heated by condensing steam at 107.8°C on the outside of the pipe wall. The steam-side coefficient has been estimated as $h_o = 10\,500\ W/m^2 \cdot K$.

(a) Calculate the convective coefficient h_i for water inside the pipe.

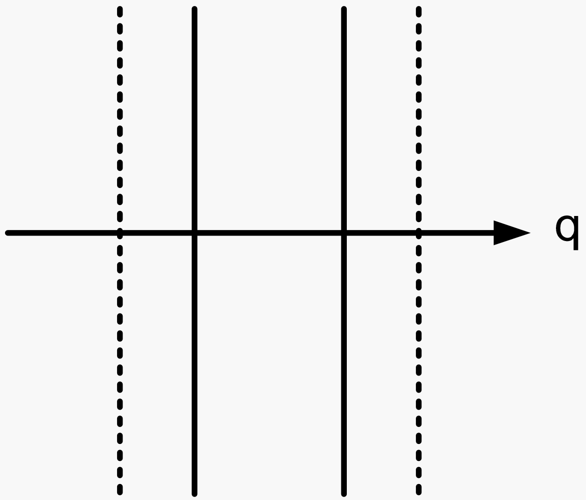
(b) Calculate the overall coefficient U_i based on the inside surface area.

(c) Calculate the heat-transfer rate q for 0.305 m of pipe with the water at an average temperature of 65.6°C.


1in schedule 40
ID=26.64mm
OD=33.4mm
k=45W/mK
Steam
T_o=107.8^{o}C
h_o=10500W/m^{2}K
water
T_i=65.6^{o}C
v=2.44m/s
h_i
k
h_o


initial guess T_w
input ε
μ_w 계산, h_i 계산
R_1, R_2, R_3 계산
$T_w = T_{wc}$
$T_{wc} = T_{water} + \frac{R_1}{\sum R}(T_{steam} - T_{water})$
$\left| \frac{T_w - T_{wc}}{T_w} \times 100 \right| < \varepsilon$
No
Yes
h_i, U_i, q 출력

4.5H 대수 평균 온도차 (log mean temp. difference ; LMTD)

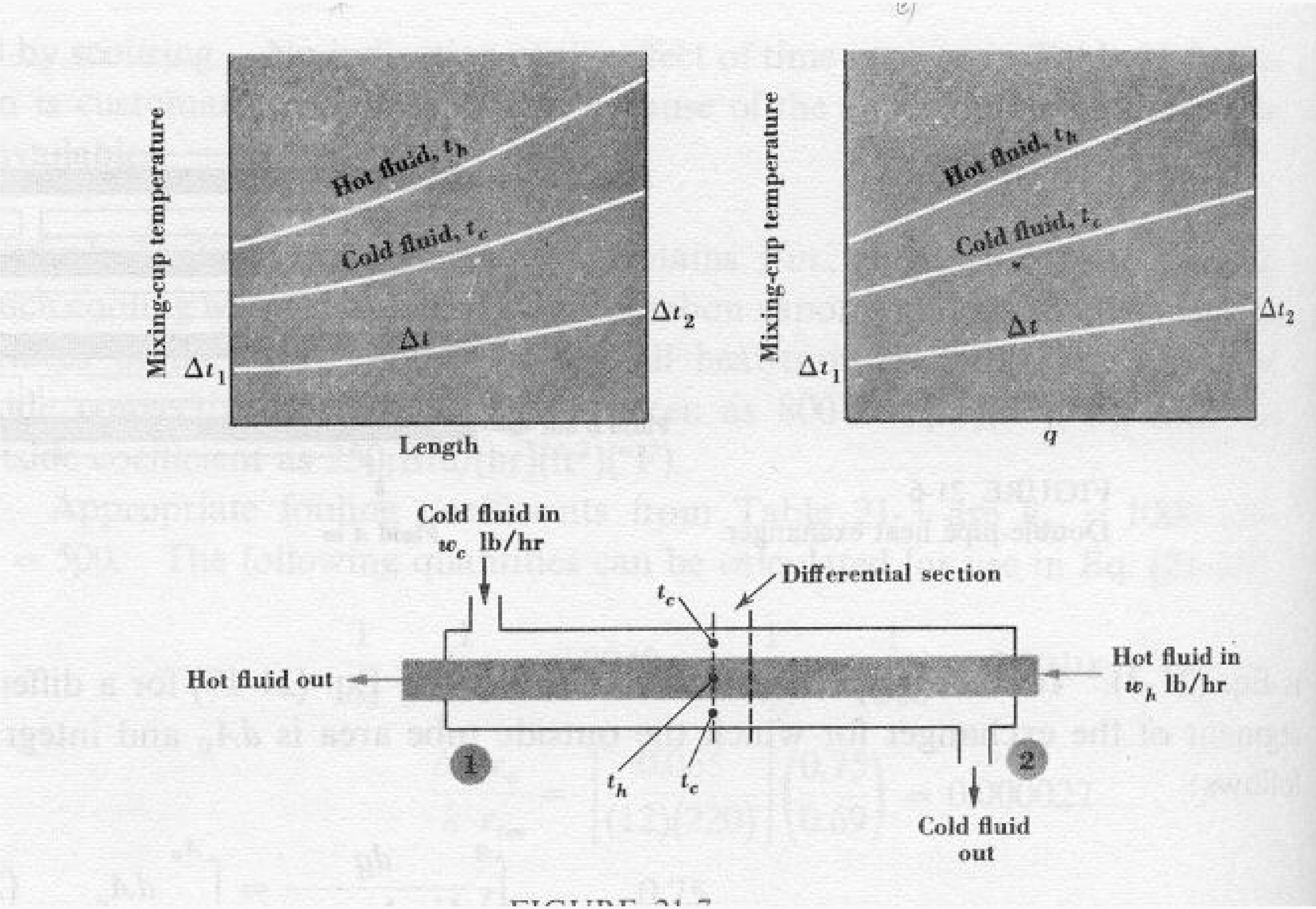


$$q = U_i A_i (T_o - T_i) = U_o A_o (T_o - T_i)$$
$$= UA\Delta T$$

ΔT가 전열면적에서 일정

열교환기에서처럼 고온유체와
저온유체가 흐르면서 열교환이
일어날 때 ΔT는 위치에 따라 변한다.

$$q = U_i A_i \Delta T_{lm} = U_o A_o \Delta T_{lm}$$



Mixing-cup temperature
Hot fluid, t_h
Cold fluid, t_c
Δt
Δt_1
Δt_2
Length
q
Cold fluid in
w_c lb/hr
Differential section
t_c
Hot fluid out
Hot fluid in
w_h lb/hr
1
2
t_h
Cold fluid out

열교환기 미소 구간에 대한 *energy balance*

$$dq = m'c'_p dT' = mc_p dT = U_o(T' - T)dA_o$$

열용량이 일정하고 각 평균온도가 앞의 그림에서

처럼 q에 대해 선형적인 관계를 가질 때

$$\frac{d(\Delta T)}{dq} = \frac{\Delta T_2 - \Delta T_1}{q}$$

$$\frac{d(\Delta T)}{U_o \Delta T dA_o} = \frac{\Delta T_2 - \Delta T_1}{q}$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{U_o \Delta T} = \frac{\Delta T_2 - \Delta T_1}{q} \int_0^{A_o} dA_o$$

$$q = U_o A_o \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = U_o A_o \Delta T_{lm}$$

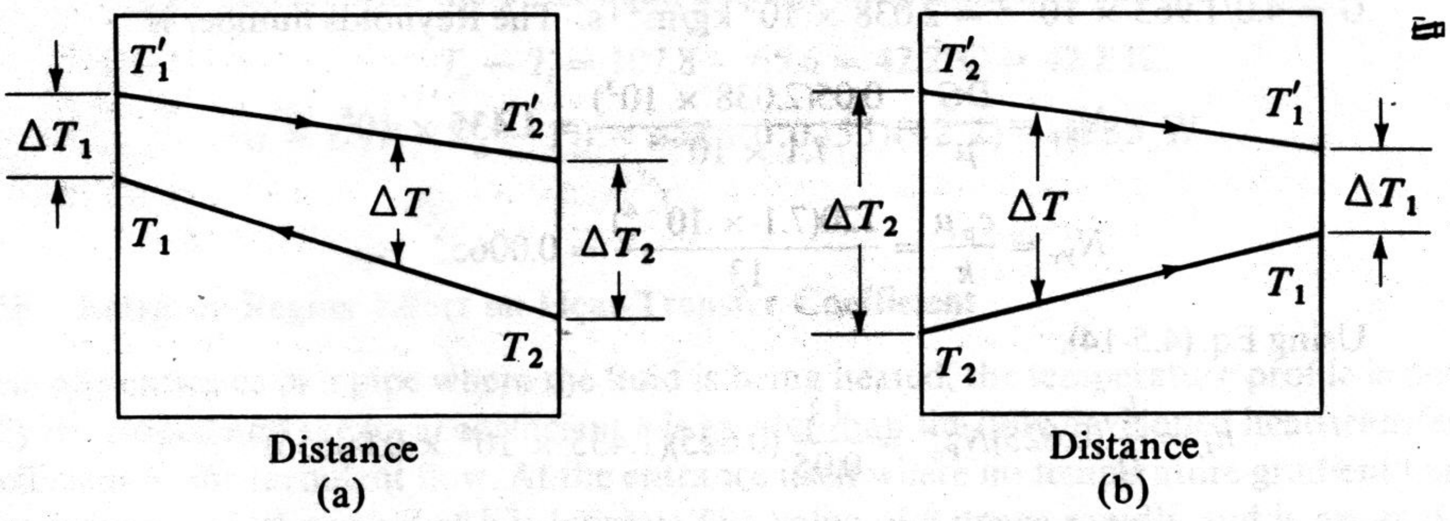


FIGURE 4.5-3. *Temperature profiles for one-pass double-pipe heat exchangers: (a) countercurrent flow; (b) cocurrent or parallel flow.*

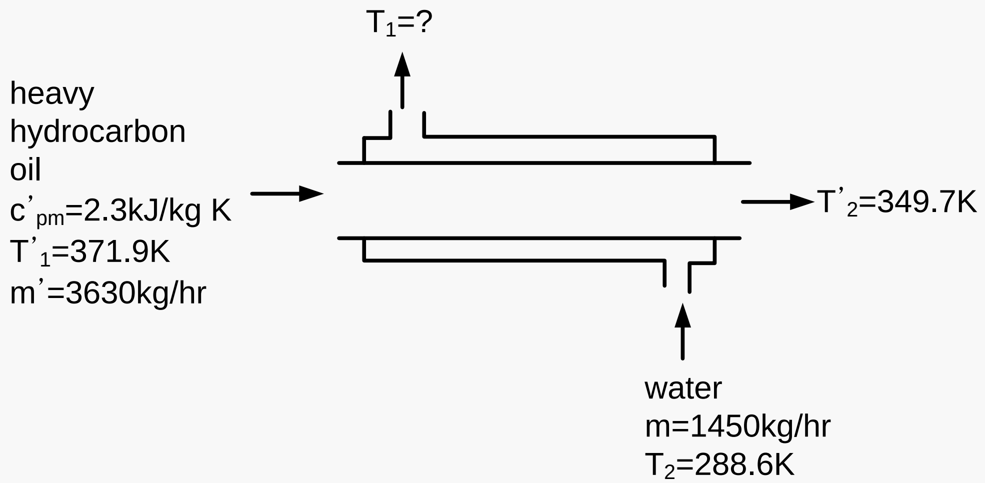
$$q = U_i A_i \Delta T_{lm} = U_i A_i \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

EXAMPLE 4.5-4. Heat-Transfer Area and Log Mean Temperature Difference

A heavy hydrocarbon oil which has a $c_{pm} = 2.30$ kJ/kg · K is being cooled in a heat exchanger from 371.9 K to 349.7 K and flows inside the tube at a rate of 3630 kg/h. A flow of 1450 kg water/h enters at 288.6 K for cooling and flows outside the tube.

(a) Calculate the water outlet temperature and heat-transfer area if the overall $U_i = 340$ W/m^2 · K and the streams are countercurrent.

(b) Repeat for parallel flow.



뜨거운 H.C.이 잃은 열량=차가운 물이 얻은 열량

$$q = m' c_p' (T_2' - T_1') = mc_p (T_1 - T_2)$$

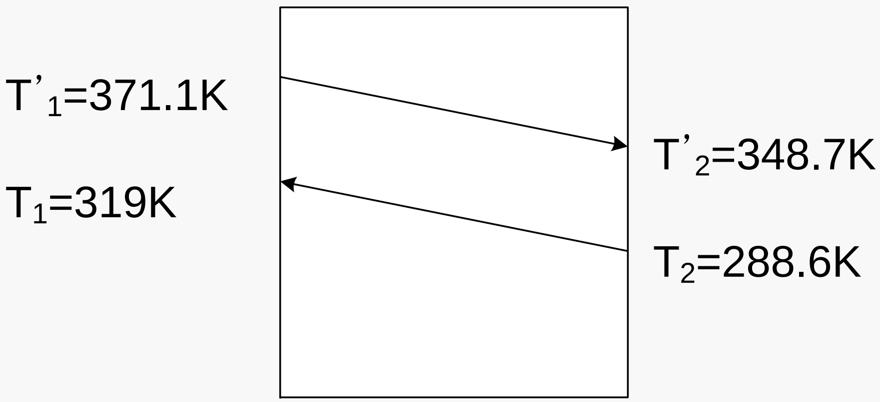
$$= \frac{3630kg}{hr}\left|\frac{2.3kJ}{kg\,K}\right|\frac{(371.9-349.7)K}{}\left|\frac{hr}{3600s}\right.$$

$$= \frac{1450kg}{hr}\left|\frac{4.2kJ}{kg\,K}\right|\frac{(T_1-288.6)K}{}\left|\frac{hr}{3600s}\right.$$

$$= 51486[kW]$$

$$T_1 = 319K$$

(a) 향류

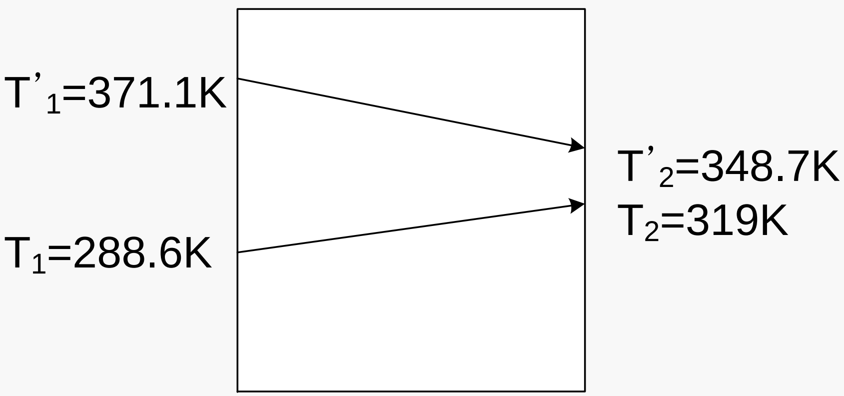


$$q = U_i A_i \Delta T_{lm} = U_i A_i \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

$$51486W = \frac{340W}{m^2 K}\left|\frac{A_i}{}\right|\frac{(61.6 - 52.9)K}{\ln(61.6/52.9)}$$

$$A_i = 2.65[m^2]$$

(b) 병류



$$q = U_i A_i \Delta T_{lm} = U_i A_i \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

$$51486W = \frac{340W}{m^2 K}\left|\frac{A_i}{}\right|\frac{(83.3 - 30.7)K}{\ln(83.3/30.7)}$$

$$A_i = 2.87[m^2]$$

4.6 HEAT TRANSFER OUTSIDE VARIOUS GEOMETRIES IN FORCED CONVECTION

침수 물체에서 평균 열전달계수

$$Nu = c\,\mathrm{Re}^m\,\mathrm{Pr}^{1/3}$$

유체의 물성은 $T_f = \dfrac{T_w + T_b}{2}$ 에서 계산

Re에서 v는 자유속도

4.6B Flow Parallel to Fluid Flow

$$Re = \frac{\rho v L}{\mu} < 3\times10^{5}\ ; laminar \quad Pr > 0.7$$

$$Nu = 0.664\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

$$Re = \frac{\rho v L}{\mu} > 3\times10^{5}\ ; turbulent \quad Pr > 0.7$$

$$Nu = 0.0366\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

4.6C Cylinder with Axis Perpendicular to Flow

$$Nu = cRe^{m}Pr^{1/3}$$

$$c, m : see\ Table\ 4.6\text{-}1$$

$$D : outside\ tube\ diameter$$

TABLE 4.6-1. *Constants for Use in Eq. (4.6-1) for Heat Transfer to Cylinders with Axis Perpendicular to Flow ($N_{Pr} > 0.6$)*

N_{Re}	m	C
1–4	0.330	0.989
4–40	0.385	0.911
$40–4 \times 10^3$	0.466	0.683
$4 \times 10^3–4 \times 10^4$	0.618	0.193
$4 \times 10^4–2.5 \times 10^5$	0.805	0.0266

4.7 NATURAL CONVECTION HEAT TRANSFER

$$Nu = f(Gr, \mathrm{Pr})$$

$$Gr = \frac{L^3 \rho^2 g \beta \Delta T}{\mu^2} = \frac{\text{부력}}{\text{점성력}}$$

$$Nu = a(Gr\,\mathrm{Pr})^m$$

a, m see Table 4.7-1

TABLE 4.7-1. *Constants for Use with Eq. (4.7-4) for Natural Convection*

Physical Geometry	$N_{Gr}N_{Pr}$	*a*	*m*	*Ref.*
Vertical planes and cylinders [vertical height $L < 1$ m (3 ft)]				
	$<10^4$	1.36	$\frac{1}{5}$	(P3)
	10^4–10^9	0.59	$\frac{1}{4}$	(M1)
	$>10^9$	0.13	$\frac{1}{3}$	(M1)
Horizontal cylinders [diameter D used for L and $D < 0.20$ m (0.66 ft)]				
	$<10^{-5}$	0.49	0	(P3)
	10^{-5}–10^{-3}	0.71	$\frac{1}{25}$	(P3)
	10^{-3}–1	1.09	$\frac{1}{10}$	(P3)
	1–10^4	1.09	$\frac{1}{5}$	(P3)
	10^4–10^9	0.53	$\frac{1}{4}$	(M1)
	$>10^9$	0.13	$\frac{1}{3}$	(P3)
Horizontal plates				
Upper surface of heated plates or lower surface of cooled plates	10^5–2×10^7	0.54	$\frac{1}{4}$	(M1)
	2×10^7–3×10^{10}	0.14	$\frac{1}{3}$	(M1)
Lower surface of heated plates or upper surface of cooled plates	10^5–10^{11}	0.58	$\frac{1}{5}$	(F1)

Assignment #7

1. 4.5-1
2. 4.5-2 (L=1m, 개별로 coding하여 Homepage로 제출)
3. 4.5-3
4. 4.5-5
5. 4.6-1