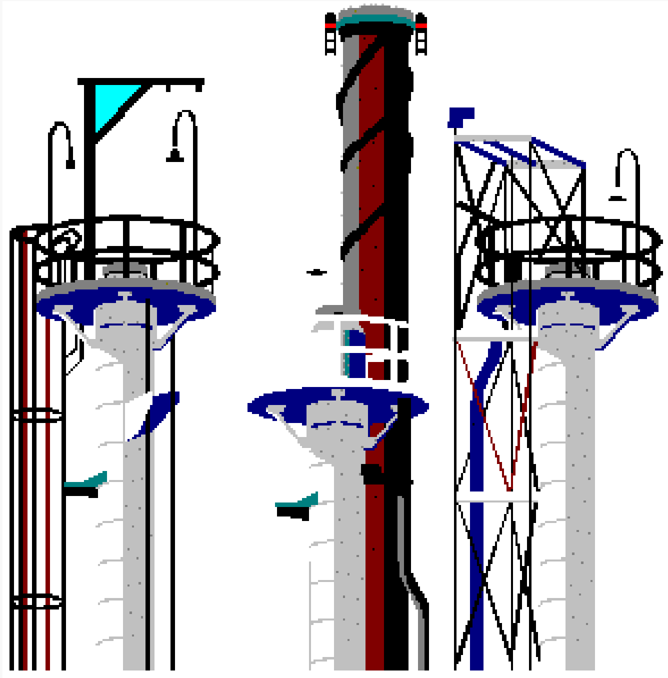
Ch2 Principles of Momentum Transfer and Overall Balance



이 광남

2.9 Shell Momentum Balance and velocity profiles in laminar Flow

► 쉘 운동량 수지

~ 쉘(미소 대상부피)에 대해 운동량 수지 적용
쉘을 미분 크기로 줄여주고
점도 법칙을 적용
속도분포, 압력에 관한 식 유도

└▶평균속도, 최대속도, 전단응력

2.9B Shell Momentum Balance inside a Pipe

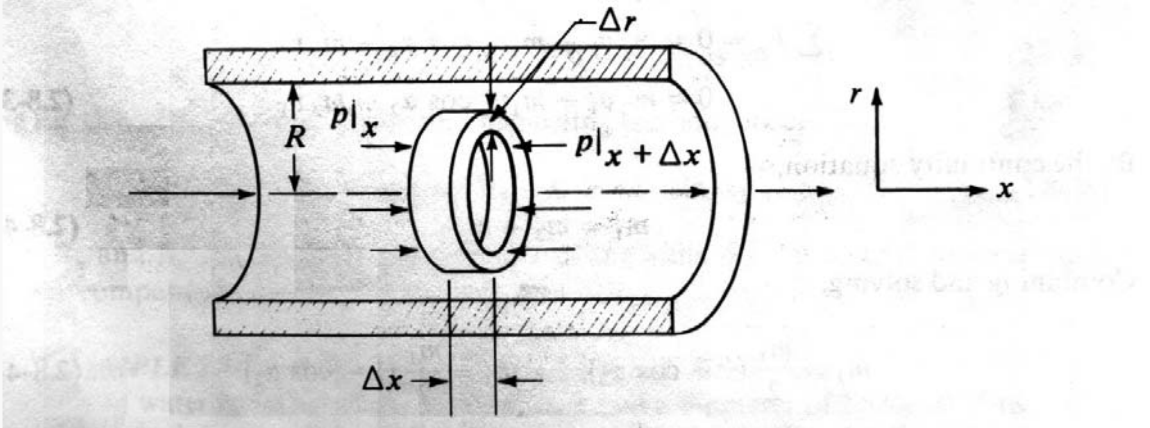


FIGURE 2.9-1. *Control volume for shell momentum balance on a fluid flowing in a circular tube.*

► Assumption

1.incompressible, Newtonian fluid

2.one-dimensional, steady state, laminar flow

3.fully developed flow

4.no-slip condition

$$\begin{Bmatrix} \text{rate of accumulation} \\ \text{of momentum} \\ \text{in system} \end{Bmatrix} = \begin{Bmatrix} \text{rate of} \\ \text{momentum in} \end{Bmatrix} - \begin{Bmatrix} \text{rate of} \\ \text{momentum out} \end{Bmatrix} + \begin{Bmatrix} \text{sum of forces} \\ \text{acting on system} \end{Bmatrix}$$

$$\rho v_x(2\pi r\Delta r)v_x\big|_x - \rho v_x(2\pi r\Delta r)v_x\big|_{x+\Delta x}$$
$$+ 2\pi r\Delta x(\tau_{rx})\big|_r - 2\pi r\Delta x(\tau_{rx})\big|_{r+\Delta r}$$
$$+ P(2\pi r\Delta r)\big|_x - P(2\pi r\Delta r)\big|_{x+\Delta x} = 0$$

$2\pi r\Delta r\Delta x$로 나누고

$$\frac{(r\tau_{rx})\big|_r - (r\tau_{rx})\big|_{r+\Delta r}}{r\Delta r} + \frac{P\big|_x - P\big|_{x+\Delta x}}{\Delta x} = 0$$

*fully developed flow*내에서는 압력구배 일정

$$\frac{P\big|_x - P\big|_{x+\Delta x}}{\Delta x} = \frac{\Delta P}{\Delta x} \rightarrow \frac{\Delta P}{L} (= \frac{P_0 - P_L}{L})$$

$\Delta r \rightarrow 0$

$$\lim_{\Delta r \rightarrow 0} \frac{(r\tau_{rx})\big|_{r+\Delta r} - (r\tau_{rx})\big|_r}{\Delta r} = (\frac{\Delta P}{L})r$$

$$\frac{d(r\tau_{rx})}{dr} = (\frac{\Delta P}{L})r$$

$$\frac{d(r\tau_{rx})}{dr} = (\frac{\Delta P}{L})r$$

$$r\tau_{rx} = (\frac{\Delta P}{L})\frac{r^2}{2} + C_1$$

$$\tau_{rx} = (\frac{\Delta P}{L})\frac{r}{2} + \frac{C_1}{r}$$

$$B.C.1 \;\; at \;\; r = 0, \;\; \tau_{rx} \neq \infty \rightarrow C_1 = 0$$

$$\tau_{rx} = (\frac{\Delta P}{L})\frac{r}{2} = (\frac{\Delta P}{2L})r$$

*momentum flux*는 r에대해 선형적으로 증가

$r = R$에서 최대값

$$\tau_{rx} = (\frac{\Delta P}{L})\frac{r}{2} = (\frac{\Delta P}{2L})r = (\frac{P_0 - P_L}{2L})r$$

$$-\mu\frac{dv_x}{dr} = (\frac{P_0 - P_L}{2L})r$$

$$\frac{dv_x}{dr} = -(\frac{P_0 - P_L}{2\mu L})r$$

$$v_x = -(\frac{P_0 - P_L}{4\mu L})r^2 + C_2$$

$$B.C.2 \;\; at \;\; r = R, \;\; v_x = 0 \;\; (no \;\; slip \;\; condition)$$

$$C_2 = (\frac{P_0 - P_L}{4\mu L})R^2$$

$$v_x = (\frac{P_0 - P_L}{4\mu L})R^2[1-(\frac{r}{R})^2] \quad parabolic \;\; velocity \;\; distribution$$

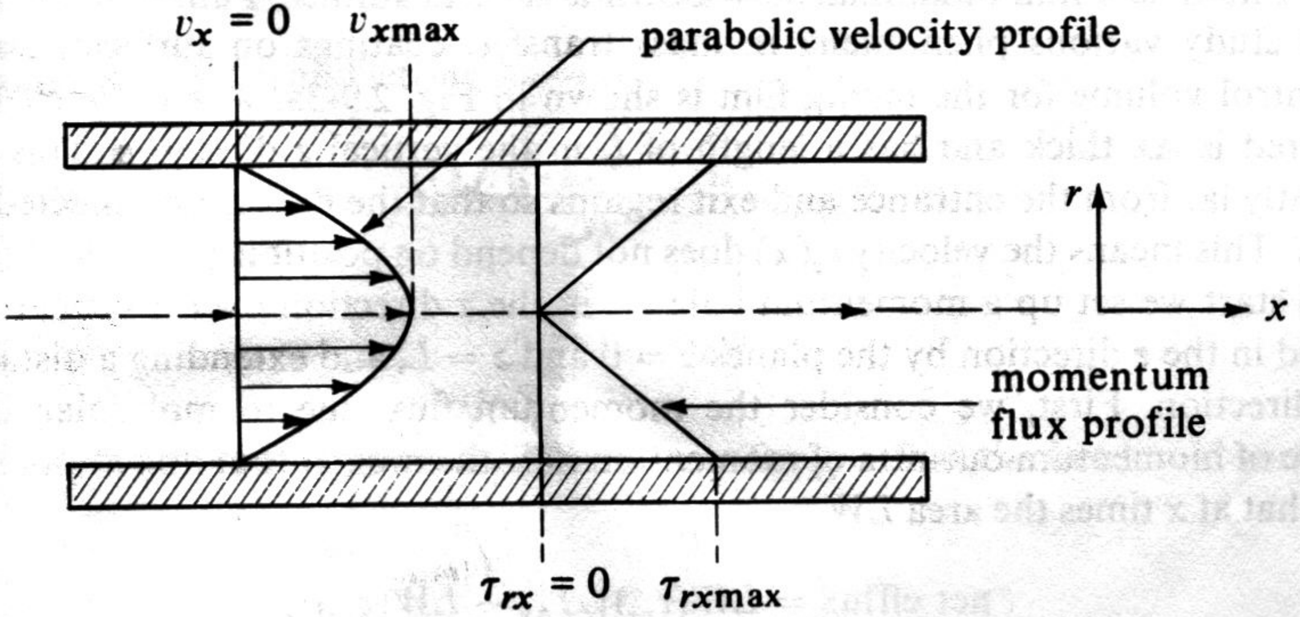


FIGURE 2.9-2. *Velocity and momentum flux profiles for laminar flow in a pipe.*

$$v_x = \left(\frac{P_0 - P_L}{4\mu L}\right) R^2 \left[1 - \left(\frac{r}{R}\right)^2\right]$$

<u>maximum velocity</u>

$$at \quad r = 0, \quad v_x = v_{x\max}$$

$$v_{x\max} = \left(\frac{P_0 - P_L}{4\mu L}\right) R^2$$

$$v_x = v_{x\max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

<u>*average velocity*</u>

$$< v_x > = \frac{1}{A}\iint_A v_x dA = \frac{v_{x\max}}{2} = \frac{(P_0 - P_L)R^2}{8\mu L} = \frac{(P_0 - P_L)D^2}{32\mu L}$$

; *Hagen – Poiseuille eqn*

$$\dot{Q} = < v_x > A = \frac{\pi (P_0 - P_L) R^4}{8\mu L}$$

$$x - component \;\; of \;\; the \;\; forces \;\; of \;\; the \;\; fluid$$
$$on \;\; the \;\; wetted \;\; surface \;\; of \;\; pipe$$

$$-R_x = (2\pi RL)(\tau_{rx})\big|_{r=R}$$

$$= (2\pi RL)(\frac{P_0 - P_L}{2L} R) = \pi R^2 (P_0 - P_L) = \frac{32\mu L\dot{Q}}{D^2}$$

$$\dot{Q} = \frac{\pi(P_0 - P_L)R^4}{8\mu L} \rightarrow \pi R^2 (P_0 - P_L) = \frac{8\mu L\dot{Q}}{R^2} = \frac{32\mu L\dot{Q}}{D^2}$$

2.9C Shell Momentum Balance for Falling Film

Shell~
길이 L
폭 W
두께 Δx

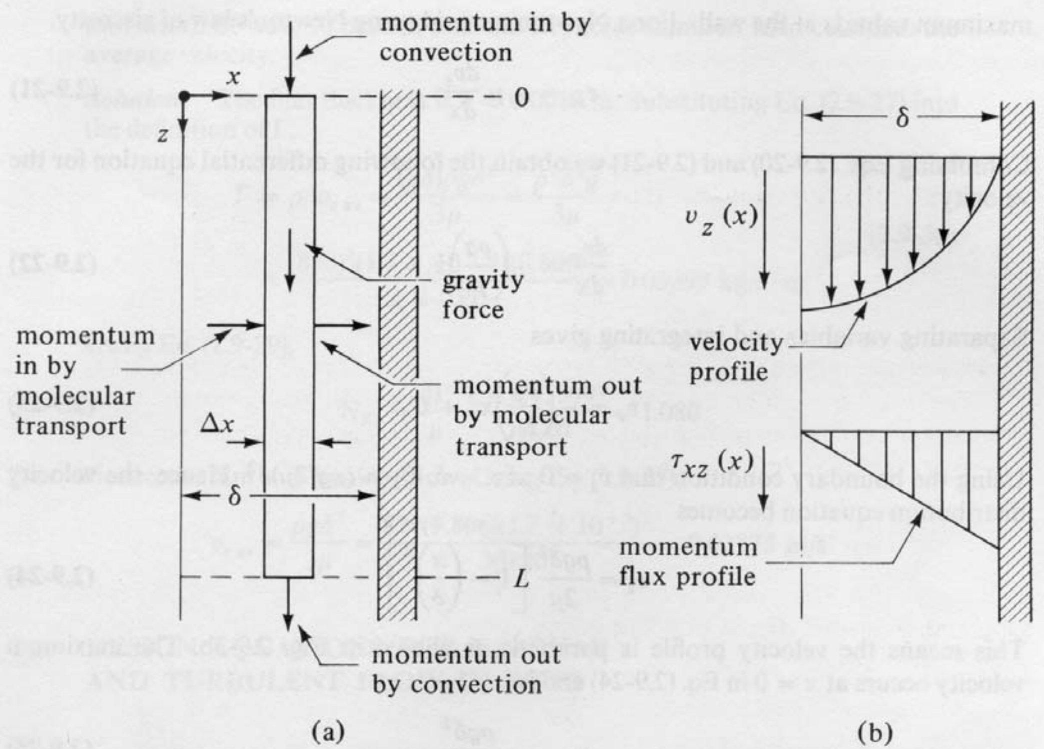


FIGURE 2.9-3. *Vertical laminar flow of a liquid film: (a) shell momentum balance for a control volume Δx thick; (b) velocity and momentum flux profiles.*

Assumption :

$$\begin{Bmatrix} rate\ of\ \ accumulation \\ of\ \ momentum \\ in\ \ system \end{Bmatrix} = \begin{Bmatrix} rate\ \ of \\ momentum\ \ in \end{Bmatrix} - \begin{Bmatrix} rate\ \ of \\ momentum\ \ out \end{Bmatrix}$$

$$+ \begin{Bmatrix} sum\ \ of\ \ \ forces \\ acting\ \ on\ \ system \end{Bmatrix}$$

$$\rho v_z (W\Delta x) v_z \big|_z - \rho v_z (W\Delta x) v_z \big|_{z+\Delta z}$$

$$+ WL(\tau_{xz})\big|_x - WL(\tau_{xz})\big|_{x+\Delta x}$$

$$+ WL\Delta x(\rho g) = 0$$

$WL\Delta x$로 나누고 $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \frac{(\tau_{xz})\big|_{x+\Delta x} - (\tau_{xz})\big|_x}{\Delta x} = \rho g$$

$$\frac{d\tau_{xz}}{dx} = \rho g$$

$$\frac{d\tau_{xz}}{dx} = \rho g \text{에서}$$

$$\tau_{xz} = \rho g x + C_1$$

$$B.C.1 \;\; at \;\; x = 0, \;\; \tau_{xz} = 0$$

$\tau_{xz} = \rho g x$ $\;\; momentum \;\; flux$ 분포는 선형

$$see \;\; Fig\, 2.9-3$$

$$-\mu \frac{dv_z}{dx} = \rho g x$$

$$\frac{dv_z}{dx} = -\frac{\rho g x}{\mu}$$

$$v_z = -\frac{\rho g x^2}{2\mu} + C_2$$

$$B.C.2 \;\; at \;\; x = \delta, \; v_z = 0 \; (no \; slip \; condition)$$

$$C_2 = \frac{\rho g \delta^2}{2\mu}$$

$$v_z = \frac{\rho g \delta^2}{2\mu}[1 - (\frac{x}{\delta})^2] \;\; parabolic \; velocity \; profile$$

maximum velocity

$$at\ \ x = 0,\ v_z = v_{z\,max}$$

$$v_{z\,max} = \frac{\rho g d^2}{2\mu}$$

average velocity

$$< v_z > = \frac{1}{A}\iint_A v_z dA = \frac{\int_0^W \int_0^d v_z dxdy}{\int_0^W \int_0^d dxdy} = \frac{W}{Wd}\int_0^d v_z dx = \frac{1}{d}\int_0^d v_{z\,max}[1 - (\frac{x}{d})^2]dx$$

$$= \frac{1}{d} v_{z\,max} \frac{2d}{3}$$

$$< v_z > = \frac{2}{3} v_{z\,max} = \frac{\rho g d^2}{3\mu}$$

volumetric flow rate

$$\dot{Q} = \langle v_z \rangle A = \langle v_z \rangle (Wd) = \frac{\rho g d^2}{3\mu}(Wd) = \frac{\rho g d^3 W}{3\mu}$$

mass flow per unit width (G)

$$G = \frac{\dot{m}}{W} = \frac{\rho \langle v_z \rangle A}{W} = \frac{\rho \langle v_z \rangle Wd}{W} = \rho \langle v_z \rangle d$$

$$Re = \frac{4G}{\mu}$$

$$Re < 1200 \quad Laminar \ \ flow$$

$z-component\ of\ the\ force\ of\ the\ fluid$

$on\ the\ surface$

$$-R_z = \int_0^L \int_0^W (\tau_{xz}\big|_{x=\delta})dydz = \int_0^L \int_0^W (\rho g \delta)dydz$$

$$= (LW\delta)\rho g\ (= mg)$$